

## **Combining Metaheuristics and Mathematical Programming for Combinatorial Optimization**

The 0–1 mixed integer programming (0–1 MIP) problem consists of maximizing or minimizing a linear function subject to equality or inequality constraints and binary choice restrictions on some of the variables. Several special cases of the 0–1 MIP problem, such as knapsack, travelling salesman and routing problems, are known to be NP-hard. Branch-and-bound and branch-and-cut methods have long been considered the methods of choice for solving mixed integer programming problems. Many contributions have led to successive improvements in these methods (S. Boussier, M. Vasquez, Y. Vimont, S. Hanafi, P. Michelon, 2010). Metaheuristic methods have attracted attention as possible alternatives or supplements to the more classical approaches (Vasquez Vimont 2005). The view adopted in this thesis is that metaheuristic approaches can benefit from a change of perspective in order to perform at their best in the MIP setting.

Recent adaptive memory and evolutionary metaheuristics for mixed integer programming have included proposals for introducing inequalities and target objectives to guide the search. These guidance approaches are useful in intensification and diversification strategies related to fixing subsets of variables at particular values, and in strategies that use linear programming to generate trial solutions whose variables are induced to receive integer values. Glover and Hanafi (2010) introduced procedures for generating target objectives and solutions by exploiting proximity in original space or projected space. In this action, we will develop more advanced approaches for generating the target objective based on exploiting the mutually reinforcing notions of reaction and resistance. Model embedded memory, as proposed in parametric tabu search, will be integrated to provide a range of recency and frequency memory structures for achieving goals associated with short term and long term solution strategies. Finally, we will propose supplementary linear programming models that exploit the new inequalities for intensification and diversification, and introduce additional inequalities from sets of elite solutions that enlarge the scope of these models. In this thesis, we propose to do an experimental study of some of the proposals in these papers. We are inclined to focus on ideas that relate to variable fixing, as in alternating intensification and diversification phases using a theme related to strongly determined and consistent variables. The studies in Hanafi and Wilbaut (2011) showed that fixing variables by the Soyster et al (1978) type of strategy could be useful, and we could go farther by using some of our more recent ideas within the strongly determined and consistent perspective.

The objective of this thesis is to design effective hybrid approaches for some combinatorial optimization problems such as: Graph Coloring, Golomb Ruler, Multidimensional Knapsack Problem.

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